

Implementation of bibliographic survey to prepare the diagnosis regarding random thinking

Implementación de una encuesta bibliográfica para preparar el diagnóstico sobre el pensamiento aleatorio.

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Abstract

Assessing educational quality in order to identify difficulties in the teaching and learning processes has been a recurring study, mainly in the competencies associated with random thinking, as they are tools that allow the formation of critical and reflective individuals of the information received from the environment. The objective of the research was to build and apply a pre-test questionnaire as a diagnosis, based on the taxonomy formed from the background review. The methodology was developed in stages, starting with the background analysis, then, with the main characteristics found here, four trends are formed: Students, teachers, curriculum and didactic material. Finally, based on the difficulties detected, a questionnaire is designed and applied to evaluate the academic levels of the tenth-grade students of the Juanambu Educational Institution, which constituted the sample under study, in terms of interpretation and construction of statistical graphs. The results showed that a high percentage of students have difficulties to read and build a statistical graph at the most complex levels: the hypothetical and the specific agreements. What is conclusive is the need to make a real and concrete diagnosis in educational institutions, in order to intervene pedagogically to improve said academic results in mathematics.

Keywords: Random thinking; Statistical graphs; Interpretation; Representation.

Implementación de una encuesta bibliográfica para la elaboración de un diagnóstico sobre el pensamiento aleatorio.

Resumen

La evaluación de la calidad educativa para identificar dificultades en los procesos de enseñanza y aprendizaje ha sido objeto de estudio recurrente, principalmente en las competencias asociadas al pensamiento aleatorio, ya que constituyen herramientas que permiten la formación de individuos críticos y reflexivos a partir de la información recibida del entorno. El objetivo de la investigación fue construir y aplicar un cuestionario pre-test como diagnóstico, basado en la taxonomía formada a partir de la revisión de antecedentes. La metodología se desarrolló por etapas, comenzando con el análisis de antecedentes; luego, con las principales características encontradas, se conformaron cuatro tendencias: estudiantes, docentes, currículo y material didáctico. Finalmente, con base en las dificultades detectadas, se diseñó y aplicó un cuestionario para evaluar los niveles académicos de los estudiantes de décimo grado de la Institución Educativa Juanambu, que conformó la muestra de estudio, en términos de interpretación y construcción de gráficos estadísticos. Los resultados mostraron que un alto porcentaje de estudiantes presenta dificultades para leer y construir un gráfico estadístico en los niveles más complejos: los acuerdos hipotéticos y específicos. Lo que se concluye es la necesidad de realizar un diagnóstico real y concreto en las instituciones educativas, con el fin de intervenir pedagógicamente para mejorar dichos resultados académicos en matemáticas.

Palabras clave: Pensamiento aleatorio; Gráficos estadísticos; Interpretación; Representación.

Introduction

In order to achieve educational quality mainly in the field of Mathematics, different international organizations propose to evaluate educational systems while maintaining a commitment to constant research for continuous improvement, such is the case of UNESCO, who since 1994 has been advancing census tests called LLECE, SERCE and TERCE, whose results indicate low academic levels in mathematics and natural sciences for Latin America, the United Nations Educational, Scientific and Cultural Organization [1], places Colombia in the regional average The Organization for Economic Cooperation and Development (OECD) also proposes comparative evaluations on an international scale such as the PISA, TIMSS tests, among others.

Thus, in Colombia educational quality has become one of the main purposes of the public policies advanced by the Ministry of National Education (Mineducación), for this it carries out monitoring with the application of SABER tests to students in different areas of the knowledge such as Language, Science, Citizenship Competencies and Mathematics. For the particular case of Mathematics, situations associated with the five mathematical thoughts are proposed and especially the random system, which has been the protagonist in terms of interpretation and representation of everyday information that comes from different social sources.

According to the curricular standards "Mathematical competences are not achieved by spontaneous generation, but rather require learning environments enriched by meaningful and comprehensive problem situations, which make it possible to advance to more complex levels of competence" [2, p. 49]. For this reason, the skills of interpretation and representation of information must be part of the mathematics curriculum organized in Educational Institutions so that their students are formed as critical and reflective citizens, capable of making decisions based on all the information they receive. of the environment where they live.

So, for [3], by being able to identify the difficulties that students present in skills such as: Interpreting and constructing statistical graphs, it will be possible to focus on the design of a methodology that contributes to improving the levels of knowledge of students in these skills.

In the case of the Juanambú Educational Institution, there is an analysis of the external tests, in terms of percentage, specifically in the learning associated with the skills of random thinking, where results that may be susceptible to improvement are generally observed. , the above is summarized in the following Table I

Table I. Average percentage of incorrect answers in learning evaluated in relation to Statistics, last six years.

Understands and transforms quantitative and schematic information presented in different forms.						Faced with a problem that involves quantitative information, propose and implement strategies that lead to appropriate solutions.					
2015	2016	2017	2018	2019	2020	2015	2016	2017	2018	2019	2020
NO	53%	33%	40%	52%	29%	NO	53%	51%	51%	60%	fifty%

Source: SABER 11th exam results report by application

To analyze the internal results of the Juanambu Educational Institution, obtained by students in the competencies associated with statistical thinking, there is still no research strategy or starting point, both to identify the specific difficulties and to characterize and have a solid base. that allows planning strategies for institutional improvement.

Due to the above, the main purpose of this research is that it is constituted as the main tool to investigate the problems related to the development of random thinking in students, specifically in the construction and interpretation of statistical data and graphs, learning that can be applied in different fields of daily life.

Subsequently, with the results of the bibliographic research, trends are formed from different aspects related to the teaching and learning processes, which go beyond presenting a report of difficulties, because the information obtained will be useful to construct a questionnaire that, when applied to the tenth grade students of the Juanambu Educational Institution, allows them to evaluate and discuss the results as a diagnosis of the academic level in the competencies associated with random thinking.

Methodology

The approach used in this study is quantitative at a descriptive level, because the aim is to collect information that can be presented with numerical data for its respective analysis. In addition, data collection was carried out directly from the investigated subjects, that is, they are primary data with a living source and the researcher obtained the information without altering the existing conditions, then, the methodological design is field, [4], to affirm this proposal we start from the definition of research of [5] "An investigation can be defined as an effort that is used to solve a problem, of course, of knowledge" (p. 47).

For the selection of the participants, the inclusion criterion was that all the students in the group were interested in participating in the activity, therefore all courses were invited, but full approval was obtained only with the fourteenth course, therefore It is concluded that the Intentional non-probabilistic sampling technique is applied, there are 28 young people between 15 and 17 years old, made up of 28 students where the socioeconomic level of the families corresponds to strata 1 and 2:

In the development of the research, several phases were considered:

First phase: Bibliographic review of the background, in order to identify the characteristics of previously carried out investigative work referring to the interpretation and construction of statistical graphs.

Second stage: Based on this bibliographical survey, obtained in the previous phase, a questionnaire was designed in which the most recurrent difficulties and suggestions about the need to implement different levels of complexity in the questions were incorporated.

Third phase: Validation by experts, since the questionnaire was constructed and adapted to the conditions of the students, it should be mentioned that this instrument was validated by three experts who work as university teachers: two of them with a master's degree and one with a doctor in education. mathematics, this before being applied to the participating students.

Fourth phase: Application of the questionnaire to students as an initial assessment to obtain a diagnosis of the academic level in the construction and reading of statistical graphs.

Results and discussion

First phase bibliographic review

For the development of this section, it was considered to analyze bibliographic material published in mathematical journals with virtual dissemination and free access, corresponding to a period of six years from 2016 to 2021. The above allowed us to find 21 articles of interest for the construction and interpretation of statistical graphs, in them 61 citations (directly from the authors) and for references (citations of citations), the original source was searched. With the results obtained from the bibliographic review, it was possible to identify and characterize four research trends: Students and their practices (T1), teachers and their practices (T2), curriculum (T3) and teaching material (T4), with common approaches in the development of the respective studies analyzed. The above is summarized in Table II.

Table II. Magazine exploration summary

Magazine name and link	Articles and quotes	Trend percentage			
		T1	T2	T3	T4
Mathematics education https://www.revista-educacion-matematica.org.mx/revista/	Total No. of items 10 Total number of citations 26	38.46	26.92	19.23	19.23
Union: Ibero-American magazine of mathematics education https://union.fespm.es/index.php/UNION	Total No. of items 6 Total number of citations 11	36.36	18.18	27.27	18.18
Relime https://www.relime.org/	Total No. of items 3 Total number of citations 8	37.5	25	12.25	12.25
Educational Thinking. Latin American Educational Research Journal	Total No. of items 1 Total number of citations 4	25	25	25	25

Science Teaching	Total No. of items 1 Total number of citations 5	T1 twenty	T2 40	T3 0	T4 40
Thesis Danilo Díaz Levicoy	No. quotes 4	T1 fifty	T2 25	T3 25	T4 0
Thesis Pedro Javier Rojas	No. quotes 3	T1 0	T2 0	T3 0	T4 100

T1 Students and their internships

In this trend, the construction and interpretation of statistical graphs is promoted in students as an important part of statistical culture [6], because in the context where the individual is trained there is always diverse information present, therefore, by not having initial training in statistical learning, there is a risk of accepting erroneous interpretations and therefore making decisions based on them will be wrong [7].

Statistical learning for students is considered as an activity of converting semiotic records, where it is important to determine the construction conventions of a graph, as stated by [8], which includes elements such as: titles and labels, frame and units. measurement and specifiers. Another sector refers to the types of statistical graphs that are most widely used in school textbooks [9], where circular sectors, line and bar diagrams, and pictograms in primary school stand out.

Regarding reading a graph according to [10], students can start by reading the statement for external identification, then determine a correspondence between variable and frequency and finish with the conclusions. From this reading of graphs, [11], [12] propose three levels: Rational level, where the student answers questions, detects trends, but does not question the information; critical level, the student understands the text without looking for hypotheses and hypothetical level, where each one forms his or her own alternative model.

T2 Teachers and their practices

For [13], the attitude of teachers towards statistics can affect the process of teaching random thinking to their students, therefore, it is observed that teachers in the urban sector have a more positive attitude towards the construction and interpretation of statistical graphs, than those in the rural sector, this may be due to the varied amount of information they view in their city environment, pressuring them to have a critical stance towards the representation and interpretation of data, in the same way [14], suggest that statistical culture of the teacher is framed in three levels of appropriation of statistical knowledge: First, basic understanding of statistical terms, second, understanding of statistical language in the context and third, the attitude of critical questioning, on the other hand, [15], highlights principles that They guide the teaching of statistics towards the interpretation of information, such as: collection and presentation of real data in tables and graphs, monitoring and discussion of work in class.

T3 Curriculum

The representation and interpretation competence associated with random thinking is part of the mandatory learning that makes up the academic curriculum of several countries [16]; This implementation, for [17], should be less technical and more practical in a real context, where the understanding of statistical information involves basic skills of construction and interpretation of statistical data and graphs, but that leads to more advanced knowledge, it is That is, priority is given to random thinking over the data system. In this same sense [18], the statistical analysis included in the curriculum contemplates three messages about variation: it can be present in all topics, it has practical consequences and is useful in a variant world.

T4 Teaching material

Given the concern raised by [19], to identify what the didactic-mathematical knowledge is to teach representation and interpretation skills in a curriculum that establishes planned learning cycles, with constant participation of the teacher and students, in response to such question, arise different investigations such as the works of [20], where dynamic interaction methodologies are proposed in study groups that improve the attitude towards statistics; [21] point out the importance of focusing on the use of problem-situations as a strategy to make sense of mathematical techniques and theories; [17], propose that learning statistics through projects promotes the evaluation of statistical knowledge, in a context of science and society.

The common aspects raised by the previous studies contemplate the need to develop a specific diagnosis that evaluates the levels of performance obtained by the students, in the competencies associated with random thinking, in order to make a didactic intervention to improve said results, such as An example of this idea is presented in Didactic Engineering [22], where in the first stages a background check must be done and then, based on those results, build and apply a questionnaire and thus make detailed planning of the other stages in the pedagogical intervention process.

Second phase: construction of the questionnaire

Since after the background documentary review it was verified that there is no instrument that particularly meets all the expected characteristics, which motivated the creation of ad hoc instruments. It should be noted that the questions are based on test models proposed in the Evaluate to Advance in Mathematics 2021 and 2022 program, of the Colombian Institute for the Evaluation of Public Education (ICFES).

A knowledge test is designed focused on two mathematical processes associated with random thinking, the construction of statistical graphs and their interpretation. Table 3 describes the general structure considered for the construction of the test based on theoretical references that allow the definition of various levels of performance.

Table III. Conventions for locating and constructing questionnaire questions

C1 Question levels [30]	C2 Construction of statistical graphs [29]	C3 Graph Interpretation (data and graph reading) [11], [12]
C1.1 Elemental level Questions related only to extracting data directly from the graph	C2.1 Mention of features Words Titles, labels and scales of the axes, keys to understanding the relationships represented	C3.1 Rational/literal level
C1.2 Intermediate level Questions related to evaluating trends based on a portion of the data	C2.2 The mathematical content Underlying: numerical sets, area of sectors, length of lines, coordinates	C3.2 Critical level Understand the context, question and evaluate the reliability of information
C1.3 Upper level Questions about deep data structure, trend comparison, and clustering	C2.3 The specific agreements Aspects specific to each graph, related to the reading, construction or correct choice of a graph	C3.3 Hypothetical Level Interpret, evaluate and formulate alternative hypotheses or models

The selection and adaptation of the questions, which make up the questionnaire applied to the students, are intended to locate the specific elements described in the previous table 3, with the purpose of facilitating the evaluation of each item. The questions were grouped into two parts, the first goes from item one to six, related to the construction of statistical graphs for three semiotic registers: Everyday language, data table and statistical graph, with their respective conversions, that is, it can go from one another with the information provided in the question, then these items require development by the student. The second part is located from item seven to twelve, corresponding to reading graphs and are multiple choice questions with a single answer, but the student must justify the selection they make. The description and purpose of each questionnaire item are summarized in Table IV.

Table IV. Summary of the description of the items

Competence	Reagent	Purpose and Description
Construction of graphs Semiotic register: Everyday language Data table Statistical graphs	Item 1. C1.2 C2.2	Conversion of two semiotic registers Go from data table to graph Information is presented in a data table about the Olympic Games and a bar diagram is requested to be constructed.
	Item 2. C1.3 C2.3	Combination of graphics identifying their specific elements Question related to citizen competencies Go from graphs to unified graph Information is presented in several graphics about school voting. Request to build a unified graph
	Item 3. C1.1 C2.1	Conversion of two semiotic registers Go from graph to data table A frequency polygon on shoe size is presented and the data table is constructed.
	Item 4. C1.2 C2.2	Conversion of two semiotic registers Go from graph to table Relationship with the environment A bar diagram is presented with information on reforestation and it is requested to build the data table

	Item 5. C1.3 C2.3	Words Labels and chart elements Question related to the students' environment An incomplete bar diagram is presented on coffee production, the information is requested to be completed
	Item 6. C1.2 C2.2	Words in graphs and conversion of two semiotic registers Go from incomplete table to graph An incomplete table is presented on the month of birth, the table must be completed and a graph constructed
Graph interpretation Multiple choice questions with only one answer and justify the selection	Item 7 C1.1 C3.1	literal reading Pictogram reading A pictogram is presented about the number of students who passed an exam, it is requested to identify numbers of approved students
	Item 8. C1.2 C3.2	Chart-Specific Conventions Bar chart reading Information is presented on monthly water consumption; it is requested to identify total values
	Item 9 C1.2 C3.2	literal reading Reading circular diagram Information about cell phone plan is presented, it is requested to analyze the percentages
	Item 10 C1.2 C3.2	rational reading Reading data table Lunch options and their prices are presented, you are asked to select the best option
	Item 11 C1.3 C3.3	Hypothetical reading Simultaneous reading of graphs and tables Information about telephone bill is presented, it is requested to identify the unnecessary option
	Item 12 C1.3 C3.3	Hypothetical reading Chart reading Comparison of brands and quantities of products is presented, a request is made to predict a possible sale

Third phase: Validation process

For the questionnaire validation process, contact is made with three professionals from the disciplinary area with extensive experience in the educational process whose profiles are: a) One has a Doctor of Education degree working as a teacher in Higher Education; b) Two with a Master's degree in Mathematics Education, but one works as a Higher Education teacher, while the other is an official teacher but with tutor functions in the Everyone to Learn Program (PTA), with whom the questionnaire along with the operationalization matrix and validation format. For two weeks these people carried out the respective review of the instrument and it was approved to be applied. With ratings of 98, 100 and 97 points on a scale of zero to one hundred points.

Fourth phase: Presentation of results

From the results obtained in the questionnaire, it is observed that for the construction of statistical graphs, 17.9% have elementary knowledge, which confirms the students' ability to make conversions between semiotic registers: graph and data table, maintaining the idea from [23], regarding the fact that for each mathematical content there must be at least two different ways of expressing and representing them.

In the questions on the construction of statistical graphs, the semiotic registers involved:

Language, data table and graphs, highlight the way in which students combine the registers using models similar to those represented in other questions, so it can be seen that their main role is not representing mathematical objects but rather working on them and with them, substituting some signs for others. To do this, the student must recognize the elements involved and understand the information presented there [24].

It should be noted that tenth grade students are familiar with the processes of tabulation and graphical representation, because it is used in natural sciences (physics and chemistry) and in mathematics when graphing the linear function from a data table. . So, we can assume that students associate previous knowledge with new learning. Prior knowledge, for [25], considers that knowledge and attitude towards statistics begins at a first level with the students' previous memories or learning.

21.4% made a scheme to locate the elements of the graph such as the bar diagram, this may be a consequence of the context in which the participating students live, the area is a coffee area and there is familiarity with the information presented, such as by using a synonym of the word production with the word coffee harvest. For [13], [26], the importance of taking into account the context is due to decision-making that is generally based on information from the environment where one lives.

78.6% of the students wrote the answer related to the words such as: Labels, Titles and scales, however, a predisposition to imitate the construction of graphs similar to those presented in the questionnaire is observed. This is where the ability to detail the elements in a graphic representation of any information is rescued. [27] provide four skills that are developed in statistical learning, the recurring one of which is modeling taken as an initial basic process.

The greatest difficulty that was detected in the students corresponds to the questions related to the reading and interpretation of information presented in statistical graphs, the relevant results 53.6% have a level of rational literal reading of texts accompanied with pictograms, a 3, 6% achieve a critical reading level by understanding the context, questioning and evaluating the reliability of the information. 100.0% of the students did not answer the questions associated with the hypothetical reading level where inference and interpretation of data that cannot be detected with the naked eye is necessary and it is necessary to consider all the elements of the text and the graph to provide an answer.

The reading levels of a graph reach their greatest complexity when hypotheses are formulated based on the information presented, [28], this appreciation is also found in [29] and [30], which relate the type of question that is can do to guide the reading of a graph, with three levels: Elementary, intermediate and higher.

Conclusions

From the bibliographic survey and the pre-test results obtained by the students, several similarities were found, such as: Firstly, the question levels proposed by [30] are related

to the data that can be identified in the text, the trends and the deep structure of the information, in this case, in the results of the questionnaire, it was observed that the way in which students develop the items depends on the interpretation they make of the text of the question, therefore, for the low results at the level Higher, the explanation may be difficulties in reading the information where it is necessary to identify and compare trends, as well as grouping various explicit or implicit data in the text of the problem situation.

Another similarity refers to the degrees of the questions associated with three levels of construction of statistical graphs, proposed by [29], ranging from the words, the mathematical content and the specific conventions for each graph, for this part, in the results of the questionnaire, the students show the relationship they make of the previous concepts already acquired, with the new learning that is being evaluated, this idea is also considered in the study proposed by [31], where the research proposes as a strategy, the exploration and readjustment of previous knowledge to begin the study of statistics.

On the other hand, the questions were also related to three levels of interpretation or reading of statistical graphs, proposed by [11], [12], ranging from literal reading at the rational level, evaluation of reliability at the critical level, until the formulation of alternative models at the hypothetical level, in the latter, the difficulty identified in the students occurs when evaluating the information and based on it formulating hypotheses, however, it is important to mention that some ease was noted in interpreting a graph, when the question included information about the context where the student lives.

Finally, when preparing a diagnosis on the interpretation and construction of statistical graphs, where the aim is to visualize the academic level of the students, it is advisable to consider that in the guiding questions of the questionnaire, prior knowledge is linked, the modeling capacity and that the information is related to the context, this in order to have a starting point, in the case of planning a future pedagogical intervention that benefits the development of random thinking in the area of mathematics.

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